

Image Processing using miniKanren

Niitsuma Hirotaka

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1 Introduction

An integral image [14] is one of the most efficient optimization technique for image processing. However an integral image is only a special case of a delayed stream[1] and memoization. This research discusses generalizing concept of integral image optimization technique, and how to generate an integral image optimized program code automatically from abstracted image processing algorithm.

In order to abstract algorithms, we forces to miniKanren. miniKanren[4] is a family of programming languages for relational programming. The book The Reasoned Schemer[8] provides a complete implementation in Scheme. The core of the language fits on two printed pages. The Scheme implementation of miniKanren is designed to be easily understood.

In this research, we use forked implementation ¹ of miniKanren from original miniKanren [5]. implementation and Racket language as the scheme language interpreter ².

2 Integral image as delayed stream and memoization

For the simplicity, let us consider a moving average [10] on one-dimensional data. This discussion is generalized to more than two-dimensional case, later.

a moving average is the unweighted mean of the previous n datum points as the following.

```
(define (moving-average-simple lst n)
  (let loop1 ((i 0) (result '()))
    (if (> i (- (length lst) n))
        (reverse result)
        (loop1 (add1 i)
              (cons
                (/ (let loop2 ((j 0) (sum 0))
                    (if (>= j n) sum
```

¹ <https://github.com/niitsuma/Racket-miniKanren/tree/recursive>

² <http://racket-lang.org/>

```

      (loop2 (add1 j)
        (+ sum (list-ref lst (+ i j)))))
    n)
  result ))))

(moving-average-simple
 '(1 2 3 4 5 6) 2)

> '(3/2 5/2 7/2 9/2 11/2)

```

A moving average can be implemented using integral image based on delayed stream as the following .

```

(define (moving-average-delayed-stream lst n)
  (define lsts (list->stream lst))
  (define (sum-helper summed lsts)
    (stream-cons
      (+ (stream-car summed) (stream-car lsts))
      (sum-helper
        (stream-cdr summed)
        (stream-cdr lsts))))
  (define summed-table
    (stream-cons 0
      (sum-helper summed-table lsts)))
  (define moving-average-stream
    (stream-map
      (lambda (x y) (/ (- x y) n))
      (stream-drop n summed-table)
      summed-table))
  (stream->list
    (stream-take
      (- (length lst) n -1)
      moving-average-stream) ))

```

However, it is difficult generalizing this technique to more than tow-dimensional case, since stream is abstraction of one-dimensional data.

In the case of integral image based on memoization, the moving average filter can be implemented as the following.

```

(require (planet dherman/memoize:3:1))
(define (moving-average-memoize lst n)
  (define/memo (summed-table m)
    (if (<= m 0) 0
      (+ (list-ref lst (sub1 m))
        (summed-table (sub1 m)))))
  (build-list
    (- (length lst) n -1)
    (lambda (m)
      (/ (-
        (summed-table (+ m n))
        (summed-table m)
        ) n))))

```

Here **define/memo** defines function **summed-table** with memoization. This technique can easily generalize to more than two-dimensional case.

Some time, these delayed stream and memoization implementation, outperforms simple integral image. For example, let us consider the case only few

Haar-like features are sampled. As is often the case with the detected object is near to its position in previous movie frame. In such case, simple average is faster than integral image, since summing over whole image contains many redundant computation. The memoization technique can reduce such redundant computation.

This strategy which selects best optimize, can be formulated as more general framework using miniKanren.

3 Delayed stream and memoization in miniKanren

Let us consider the recent imprimentation of miniKanren[5]. miniKanren can be regarded as delayed stream of list of goals [9]. This raggedness becomes more clear by renaming some code blocks as more appropriate name in `lambdaf@` and `case-inf` macro as the following.

```
(define-syntax lambdaf@
  (syntax-rules () ((- () e) (delay e))))

(define-syntax case-inf
  (syntax-rules ()
    ((- e () e0) ((f^ e1) ((a^ e2) ((a f) e3)))
      (let ((a-inf e))
        (cond
          ((not a-inf) e0)
          ((promise? a-inf) (let ((f^
                                   (force a-inf))) e1))
          ((not (and (pair? a-inf)
                     (procedure? (cdr a-inf))))
           (let ((a^ a-inf) e2))
           (else (let ((a (car a-inf)) (f (cdr a-inf)))
                     e3)))))))
```

Note to lines including `delay`, `force` and `promise?`. These lines show that miniKanren evaluates list of goals as delayed stream.

miniKanren also have auto memoization mechanism. miniKanren uses triangular substitutions[?]. Triangular substitutions can be regarded as generalized memoization mechanism.

Using the following loop statement `builde` in miniKanren, these delayed stream and memoization mechanism can be included automaticaly.

```
(define (builde n f)
  (let loop ([m 0])
    (if (>= m n)
        succeed
        (fresh ()
          (f m)
          (loop (add1 m)))))
```

`builde` is the correspondence of `build-list` statement in the scheme language to miniKanren. The following example usage shows clearly its usage.

```

(let ([vs (build-list 3 var)])
  (run* (q)
    (build
      3
      (lambda (i)
        (== (list-ref vs i) i)
      ))
    (== q vs))
  )
> '((0 1 2))

```

By using **build**, moving average filter with the strategy which selects best optimize can be described as the following.

```

(define at list-ref)
(let* (
  [n 5]
  [v (build-list n (lambda(x) (random n)))]
  [t (build-list n var)]
  [r (build-list n var)]
  [s 2]
)
  (run* (q)
    (== (at t 0) 0)
    (build n
      (lambda (i)
        (fresh (t1) (== t1 (at t i))
          (project (t1)
            (== (at t (add1 i))
              (+ t1 (at v i)))))))
    (build
      (- n s -1)
      (lambda (i)
        (let ([u (build-list (add1 s) var)])
          (fresh (t1 t2)
            (== t1 (at t i) )
            (== t2 (at t (+ i s)) )
            (conda
              [
                (unified-varo t1) (unified-varo t2)
                (project (t1 t2)
                  (== (at r i) (- t2 t1)))
              ]
              [
                (fresh ()
                  (== (at u 0) (at v i))
                  (build
                    s
                    (lambda (j)
                      (if (= j (sub1 s))
                        (== (at r i) (at u (sub1 s)))
                        (fresh (x)
                          (== x (at u j))
                          (project (x)
                            (== (at u (add1 j))
                              (+ x (at v (+ i j 1))))
                        )
                    )
                  )
                )
              ]
            )
        )
      )
    )
  )

```

```

))))))
]
))))
(== q '(,v ,t ,r))
))

```

First, random signal **v** with length **n** is generated. Then the signal **v** is averaged over with window length **s**. The averaged result is stored to list **r**. List **t** is integral image of signal **v**. The first loop using **builde** construct integral image. The moving averages are computed in the second **builde** loop. Most important part is inside **conda**. There are two blocks inside **conda**. The first block calculates moving average using integral image when integral image value is already computed. **unified-varo** statement judges whether or not the value is already computed. When the value not exist, the second block evaluated. The second block simply computes moving averages using **conda** loop inside the second block.

Our proposition is the following framework.

1. describe algorithm using **builde** loop.
2. when describing algorithm, enumerate various possible relations.
3. sort the various relations with sort order its short cut length.
4. enumerate the sorted relation in **conda**
5. then, breast-first search in miniKanren chooses best optimize strategy

This framework provides generalization of integral image optimization strategy.

4 Nested loop

builde can define one-dimensional loop in miniKanren within delayed stream and memoization mechanism. Any dimensional nested loop Based on **builde** can define using the following function.

```

(define (builde-nest n-list f)
  (let loop ([n-list n-list] [i-list '()])
    (if (null? n-list)
        (apply f (reverse i-list))
        (let ([m (car n-list)])
          (builde
            m
            (lambda (i)
              (loop (cdr n-list) (cons i i-list))))))))

```

The following example usage shows clearly its usage.

```

(let ([vs (build-list 6 var)])
  (run*
    (q)
    (build-nest
      '(2 3)
      (lambda (i j)
        (==
          (list-ref vs (+ (* i 3) j ) )
          (+ (* i 3) j ))
        ))
      (== q vs)
    ))
  > '((0 1 2 3 4 5))

```

Note that, **build-nest** function can convert any nested loop into delayed stream. There is a possibility that appropriate optimization techniques can translate the converted delayed stream into integral image optimized code. However this research forces to abstraction of nested loop. Developing optimization compiler is future work.

5 Application to connected-component labeling

This section shows example application of **build-nest** to image processing application. For the example, we consider Connected-component labeling [6]. Implementing Connected-component labeling in C language need more than 20 lines.

The following is an example code taken from <http://stackoverflow.com/questions/14465297/connected-c>

```

.
int dx[] = {+1, 0, -1, 0};
int dy[] = {0, +1, 0, -1};
int row_count;
int col_count;
int m[MAX][MAX];
int label[MAX][MAX];
void dfs(int x, int y, int current_label) {
  if (x < 0 || x == row_count) return;
  if (y < 0 || y == col_count) return;
  if (label[x][y] || !m[x][y]) return;
  label[x][y] = current_label;
  for (int direction = 0;
       direction < 4; ++direction)
    dfs(x + dx[direction],
        y + dy[direction],
        current_label);
}
void find_components() {
  int component = 0;
  for (int i = 0; i < row_count; ++i)
    for (int j = 0; j < col_count; ++j)
      if (!label[i][j] && m[i][j])
        dfs(i, j, ++component);
}

```

miniKanren with **build-nest** enables implementing with one of third lines.

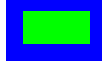


Figure 1: input image

```
(..0 ..0 ..0 ..0 ..0 ..0 ..0 ..0
 ..0 ..1 ..1 ..1 ..1 ..1 ..1 ..0
 ..0 ..1 ..2 ..2 ..2 ..2 ..1 ..0
 ..0 ..1 ..2 ..2 ..2 ..2 ..1 ..0
 ..0 ..1 ..1 ..1 ..1 ..1 ..1 ..0
 ..0 ..0 ..0 ..0 ..0 ..0 ..0 ..0 )
```

Figure 2: connected-component-labeling result using miniKanren

```
(define result
  (map var (make-list (* width height) 1)))

(run* (q)
  (build (list (sub1 height) (sub1 width) 2 2)
    (lambda (i j di dj)
      (if (equal?
            (img-ref img i j)
            (img-ref img (+ i di) (+ j dj)))
          (==
            (img-ref result i j)
            (img-ref result (+ i di) (+ j dj)))
          succeed)))
    (== q result)))
```

Figure 2 shows this programs output when the image in figure 1 is inputed. This code example shows our framework enables quite readable and concise program, especially the algorithms include many nested loop.

6 Discussion

In section 2, `moving-average-simple` function is optimized to `moving-average-delayed-stream` and `moving-average-memoize` using an integral image. The ultimate aim of this research is automatic generating these optimized code from simple code. However, the proposed framework is difficult to generete such optimized code. In order to atuomatic optimize any program, the system requires problem specic knowledge. It is hard to analyse problem specic knowledge automatically. Instead, the proposed framework selects the best relation from already enumerated relations. When human programmer enumerates relations, problem specic knowledge is analyzed by human programmer. If this enumerating process can be atomized, we can fully automatize this optization process.

Speeded up robust features (SURF) [3] is optimized algorithm from scale-invariant feature transform (SIFT) [11] with appropriate approximation. Approximation is also important factor for integral image optimization. Also It is hard to analyze approximation automatically. The proposed framework avoid this automatically analyze by enumerating relations. The analyze is done when human enumerate relations.

Many algorithms for image processing are based on raster scan over 100×100 pixels. Since relational programming languages are slow, relational programming languages are not suitable for deling with many pixels directory. Instead, relational programming languages can use for describing relation among extracted features [2]. miniKanren is one of the such slow relational programming languages. miniKanren is based on delayed stream for list of goals. Note that we can stop delayed evaluation some intermediate point. Then we can evaluate the rest evaluation by more optimized Scheme compiler. In this case, miniKanren works as prepossess optimization.

Recently many Scheme compilers including various optimization techniques have been implemented [12, 13, 7] . Sometimes, automatic application of these various optimization techniques to C programs outperformed hand-written C programs [13] . These optimized compilers also enables relational programming languages to deling with many pixels directory.

7 Conclusion

Automated optimization using an integral image is discussed. However, this research can not achieve fully automation. Instead the optimization problem is translated into enumerating relations using miniKanren. The translation based on the alalysis which an integral image is only a special case of a delayed stream and memoization. The proposed framework sometime outperform simple integral image implementation.

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